

## The Implementation of Meshless Local Petrov Galerkin (MLPG) Method for Determine Pollutant Sources in Brantas River

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### Abstract

Pollution in the river often occur and can be threaten for aquatic organisms and humans. Polluted river has negative impacts for people around of the Brantas River. Pollutants entering into the river can be derived from industrial and nonindustrial. Based on these problems, the authors conducted a study and analyze the location of pollutant sources by mathematical approach. To find the location of the pollutant sources is not easy, with applying the Meshless Local Petrov Galerkin method (MLPG) we can determine the distance of the point source of pollutant from sample point monitoring Perum Jasa Tirta. The MLPG method does not use a grid that can be used for domains that are not continuous or move. Pollutant source location obtained from the simulation results that the position of 700m to 1000m which is the fluctuation of the position of the concentration of COD, BOD, DO and TSS were lower. This indicates that there is contamination in the highest position of 700 meters until to 1000meters.

**Keywords:** *MLPG, concentration, pollutant*

### 1. Introduction

Pollutants which result in decreased oxygen levels can be detected its presence by looking at several water quality parameters such as COD, BOD, and DO. Pollutants are not dissolved by water will settle to the bottom and attached to the wall of the river and cause sediment formation. In this research we use Meshless Local Petrov Galerkin Method for determine pollutant sources in the confluences two river in Brantas river.

### 2. The Meshless Local Petrov Galerkin Method

The MLPG method does not use mesh, It is well in interpolation function with trial and test as well as in the calculation integral. This method was researched by Atlury, He get accurate and actual field conditions (see[3]).

The following linear Poisson equation is given:

$$\nabla u^2(x) = p(x), \quad x \in \Omega$$

with  $p$  is a source function, domain  $\Omega$  bounded by boundary  $\Gamma = \Gamma_u \cup \Gamma_q$ , with boundary condition as follow:

$$u = \bar{u} \quad \text{pada } \Gamma_u \quad (2.15a)$$

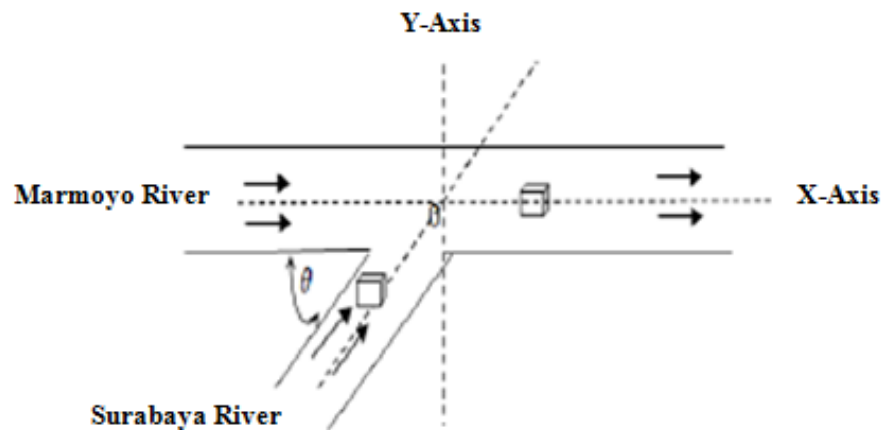
$$\frac{\partial u}{\partial n} = q = \bar{q} \quad \text{pada } \Gamma_q \quad (2.15b)$$

with  $\bar{u}$  is initial condition and  $\bar{q}$  is boundary condition. local weak form from poisson equation as follow:

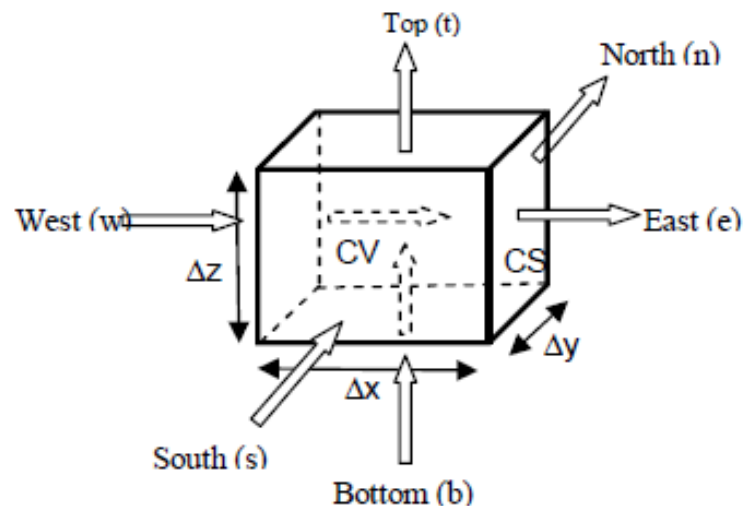
$$\int_{\Omega_s} (\nabla u^2(x) - p)v \, d\Omega = 0$$

### 3. The Mathematical Model of Brantas River

We have assumed the direction of flow of the confluence of two rivers with X-axis and Y-axis. In the area the confluence of two streams is assumed there a volume control.



**Figure 1** The Schematics of Confluence of Two River



**Figure 2** The Control Volume in Confluence of Two River

#### The Governing Equations:

##### a. Marmoyo River

Conservation of mass (Continuity Eq.):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Conservation of Momentum:

X-Axis:

$$\frac{d}{dt}(u + q_b) + \frac{d(u^2 + \frac{1}{2}gh)}{dx} + \frac{d(vu + \frac{1}{2}gh)}{dy} - gh(S_x + S_y) = 0$$

Y-Axis:

$$\frac{d}{dt}(u + q_b) + \frac{d(vu + \frac{1}{2}gh)}{dx} + \frac{d(v^2 + \frac{1}{2}gh)}{dy} - gh(S_x + S_y) = 0$$

Transport of Pollutant:

$$\frac{dC}{dt} + \frac{d(uC)}{dx} + \frac{d(vC)}{dy} = \frac{s}{\rho}$$

### b. Surabaya River:

Conservation of mass (Continuity Eq.):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \cos \theta + \frac{\partial v}{\partial y} \sin \theta = 0$$

Conservation of Momentum:

X-Axis:

$$\frac{d}{dt}(u + q_b) + \frac{d(u^2 + uv \cos \theta + \frac{1}{2}gh(1 + \cos \theta))}{dx} + \frac{d((uv + \frac{1}{2}gh) \sin \theta)}{dy} - gh(S_x + S_y) = 0$$

Y-Axis:

$$\frac{d}{dt}(v + q_b) + \frac{d(uv + v^2 \cos \theta + \frac{1}{2}gh(1 + \cos \theta))}{dx} + \frac{d((v^2 + \frac{1}{2}gh) \sin \theta)}{dy} - gh(S_x + S_y) = 0$$

Transport of Pollutant:

$$\frac{dC}{dt} + \frac{d(uC)}{dx} + v \left( \frac{dC}{dx} \cos \theta + \frac{dC}{dy} \sin \theta \right) = \frac{s}{\rho}$$

### c. Confluence of River:

Conservation of Mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = Q_1 + Q_2$$

with

$Q_1$ : Flow rate from marmoyo river

$Q_2$ : Flow rate from surabaya river

Conservation of Momentum:

X-Axis:

$$\frac{d}{dt}(u + q_b) + \frac{d(u^2 + \frac{1}{2}gh)}{dx} - gh(S_x + S_y) - v_1 Q_1 - v_2 Q_2 = 0$$

Y-Axis:

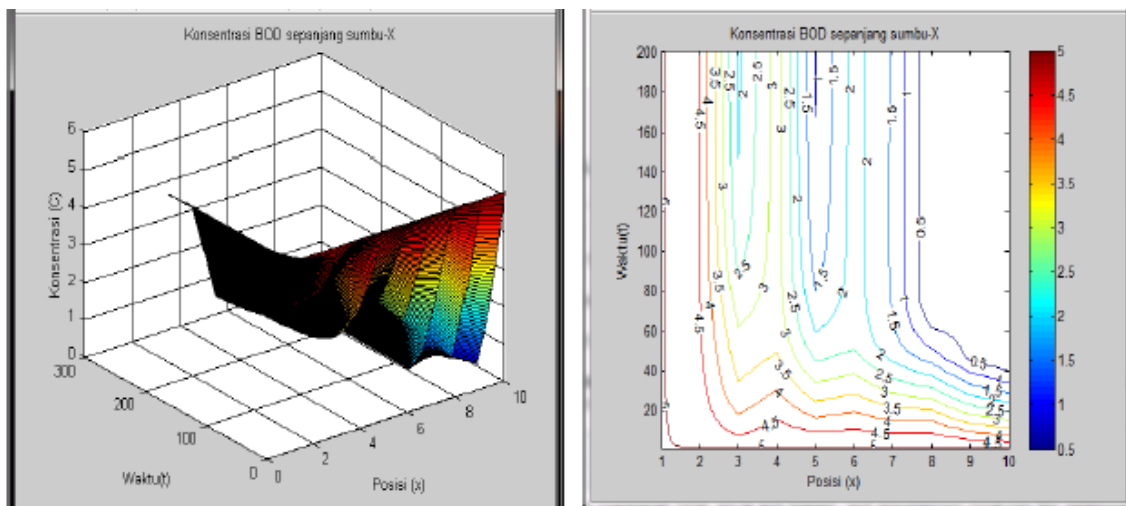
$$\frac{d}{dt}(v + q_b) + \frac{d(v^2 + \frac{1}{2}gh)}{dy} - gh(S_x + S_y) = 0$$

#### 4. Result and Simulation

##### Positioning the X-axis Direction:

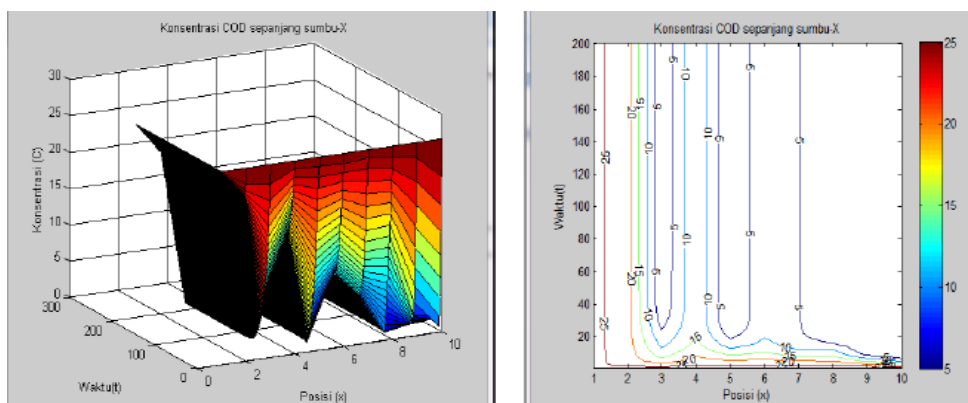
For determine the location of the pollutants sources that come from the direction along the marmoyo based on fluctuations in the concentration of COD, BOD and TSS. with provide the initial conditions marmoyo stream velocity, velocity surabaya river, COD, BOD, and TSS same in all positions (x, y). in This simulation assumed a depth of 3m and h is the time t is 10 second. Determining the location of pollutant sources are assumed position 1 represents a distance of 100m of sampling sites by Perum Jasa at Perring Bridge.

The result output of program:



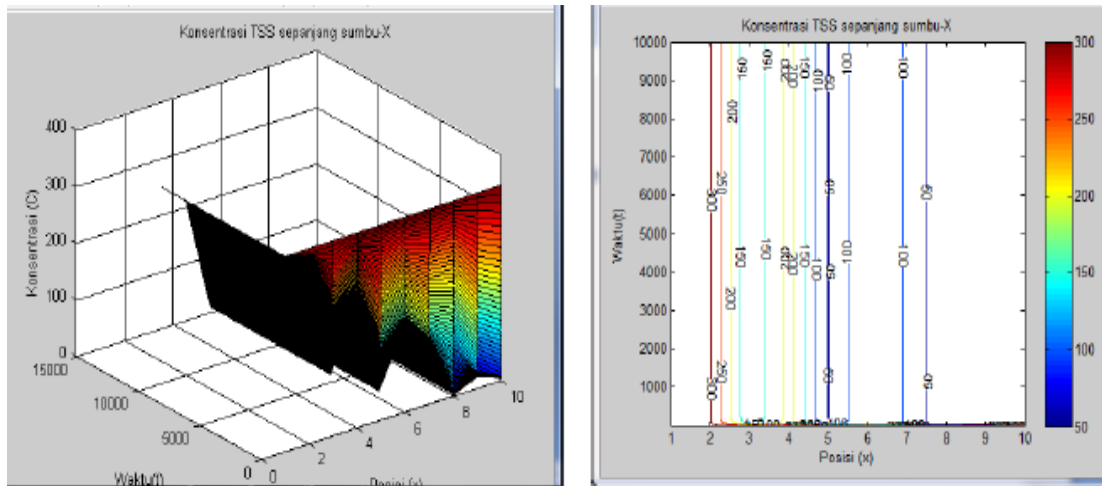
**Figure 3** Distribution BOD in X-Axis

Based on the simulation of BOD concentrations is lowest that obtained at intervals position in [7,10]



**Figure 4** Distribution COD in X-Axis

Based on the simulation of COD concentration is lowest that obtained at intervals position [2,4], position [4,6], and position [7,10].



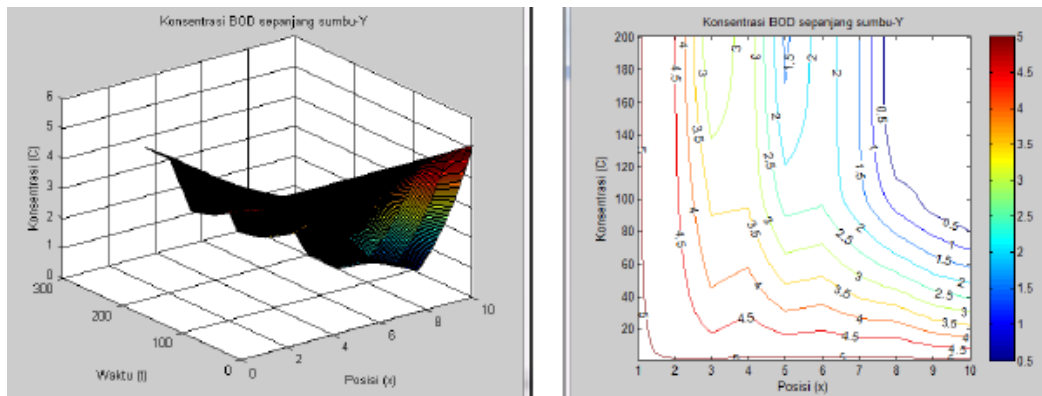
**Figure 5** Distribution TSS in X-Axis

Based on the simulation of TSS concentrations is lowest that obtained at intervals position in [5,6] and [7,10].

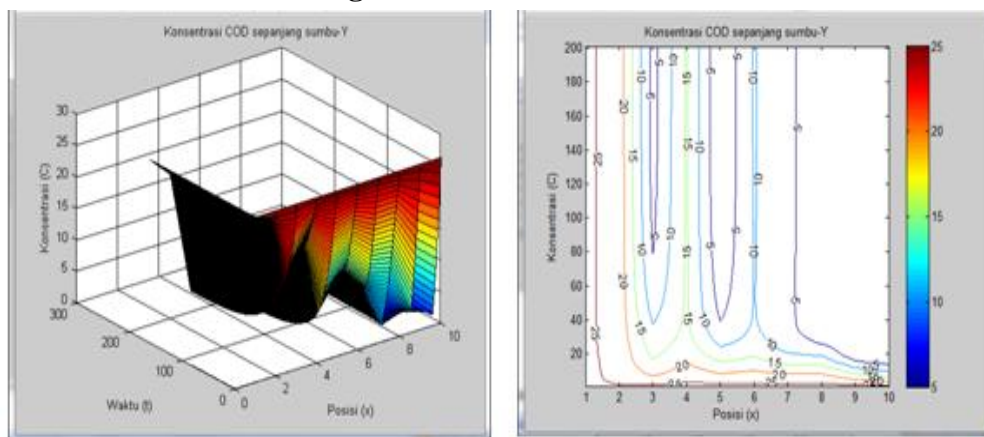
From the simulation results of COD, BOD and TSS, it is known that lowest concentration is in the interval position [7,10]. When we overlay before confluences of two river along the Marmoyo river seen that the fluctuations in the concentration of COD, BOD and TSS are increased, it is indicates that there is contamination in the highest position [7,10], which is about 700m to 1000m of sampling sites Perum Jasa Tirta.

#### Position Determination In Direction Y-axis:

Determining the location of the source of pollutants that come from the direction along the Surabaya based on fluctuations in the concentration of COD, BOD and TSS. with provide the initial conditions marmoyo stream velocity, velocity surabaya river, COD, BOD, and TSS same in all positions (x, y). in This simulation assumed a depth of 3m and h is the time t is 10 second. Determining the location of pollutant sources are assumed position 1 represents a distance of 100m of sampling sites by Perum Jasa Tirta at Pening Bridge. The following is the program output:

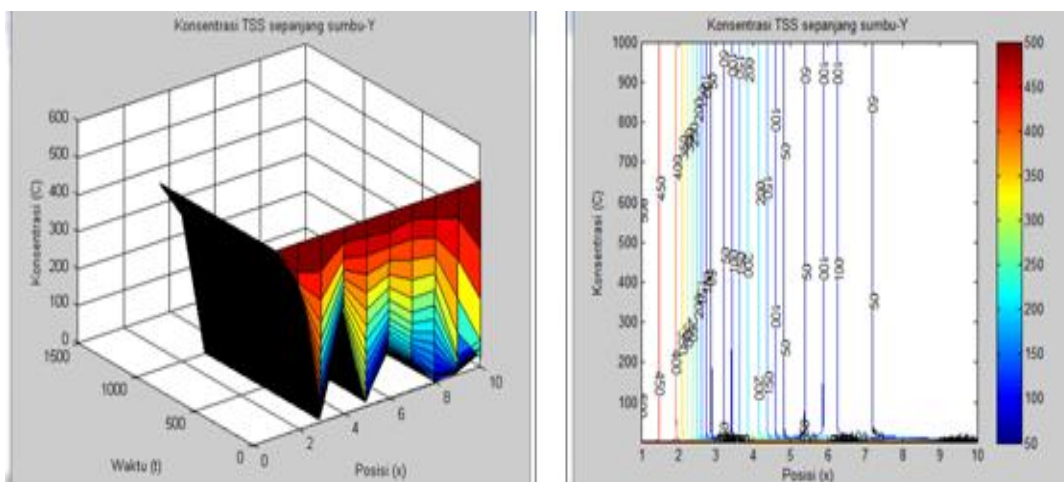


**Figure 6** Distribution BOD in Y-Axis



**Figure 7** Distribution COD in Y-Axis

Based on the simulation of low COD concentration is obtained at the position of the interval [2,3], [5,6], and [7,10].



**Figure 8** Distribution TSS in Y-Axis

Based on the simulation of the lowest TSS concentrations obtained at the position of the interval [3,4], [5,6], and [7,10]. The simulation results can be seen in appendix B.

From the simulation results of COD, BOD and TSS, it is known that changes is the lowest concentration at the position of the interval [7,10]. If this position in overlay stricken before the confluence of the river along the Marmoyo seen that fluctuations in the concentration of COD, BOD and TSS are lower, it indicates that the highest contamination in the position [7,10], which is about 700m to 1000m from the location Perum Jasa sampling.

## 5. Conclusion

Based on the simulation results it can be concluded that the fluctuation of COD, BOD, and TSS is lowest at position 7 to 10, which is at a distance of about 700 meter to 1000 meter from the sampling site Perum Jasa Tirta. It is indicated that there is a high level of pollution.

**Acknowledgements** The author would like to thanks to Prof.Dr. Basuki Widodo,M.Sc as supervisor, Perum Jasa 1 Malang, and her colleagues for their valuable comments and input.

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